Student Number___________

MORIAH COLLEGE

Year 12 – Task 2 - Pre-Trial

MATHEMATICS
2009

Time Allowed: **3 hours**

Examiners: C. White, L. Bornstein

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question

Total marks – 120

- Attempt Questions 1–10
- All questions are of equal value

Question 1. (12 Marks) Use a SEPARATE Answer Sheet. **Marks**

(a) Evaluate correct to 3 significant figures: **2**

$$\frac{e^3 + 2.58}{\sqrt{3.5 - 2.3}}$$

(b) Solve $|x - 5| \le 7$ **2**

(c) Solve $x^2 - 3x - 10 = 0$ **2**

(d) Differentiate $x^6 - 2x^{-1} + 3$ with respect to x **2**

(e) Find integers a and b such that $\left(4 + \sqrt{3}\right)^2 = a + b\sqrt{3}$ **2**

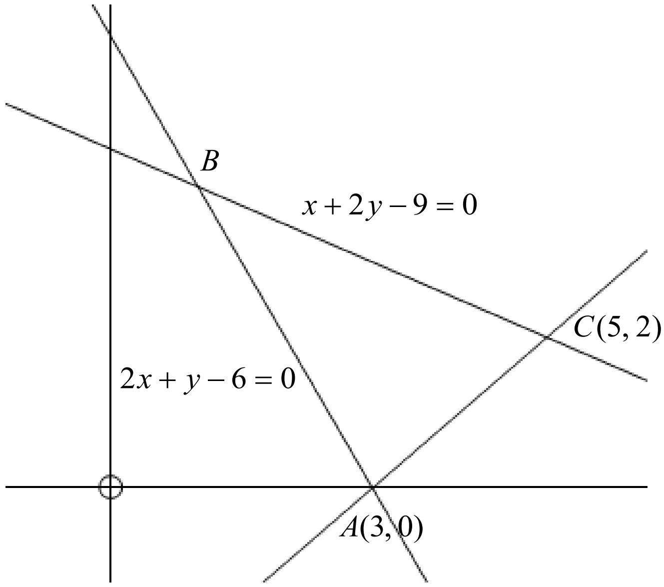
(f) Solve the following pair of simultaneous equations: **2**

$$3x + 2y = 14$$
$$2x - 4y = -12$$

Question 2. (12 Marks) Use a SEPARATE Answer Sheet. **Marks**

(a) Find the equation of the normal to the curve $y = x^2 + 5x$ at the point $(1, 6)$. **3**

(b)



The points $A(3, 0)$, B and $C(5, 2)$ form a triangle, as shown in the diagram. The line BC has equation $x + 2y - 9 = 0$ and the line AB has equation $2x + y - 6 = 0$.

(i) Find the gradient of the line AC. **1**

(ii) Show that the equation of the line AC is $x - y - 3 = 0$. **1**

(iii) Show that the coordinates of the point B are $(1, 4)$. **2**

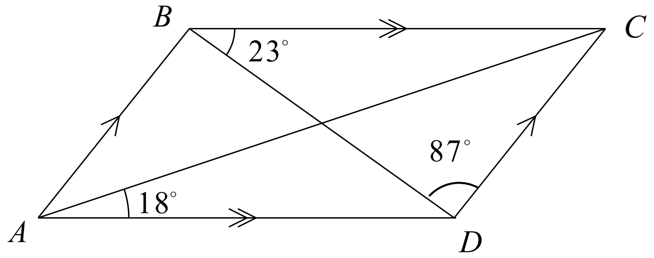
(iv) Show that the perpendicular distance between the point B and the line AC is $3\sqrt{2}$. **2**

(v) Find the area of the triangle ABC. **3**

Question 3. (12 Marks) Use a SEPARATE Answer Sheet. **Marks**

(a) Evaluate $\int_1^2 \left(\frac{3x^4 - 2x}{x^3}\right) dx$ **2**

(b) **2**



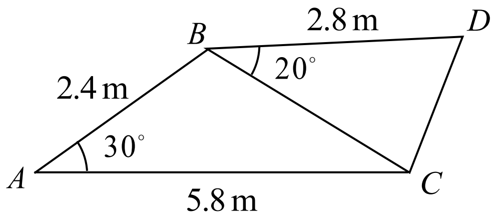
In the diagram $ABCD$ is a parallelogram. Given that $\angle CBD = 23^\circ$, $\angle CAD = 18^\circ$ and $\angle BDC = 87^\circ$, find the size of $\angle ACD$. Give reasons for your answer.

(c) Differentiate with respect to x:

(i) $(x^2 - 3)^7$ **2**

(ii) $\frac{2x}{e^x}$ **2**

(d)



NOT TO SCALE

In the diagram above $AB = 2.4\,\text{m}$, $AC = 5.8\,\text{m}$, $BD = 2.8\,\text{m}$, $\angle BAC = 30^\circ$ and $\angle DBC = 20^\circ$.

(i) Show that the length of $BC = 3.9\,\text{m}$, to 1 decimal place. **2**

(ii) Hence or otherwise find the area of triangle BCD, to the nearest metre. **2**

Question 4. (12 Marks) Use a SEPARATE Answer Sheet. **Marks**

(a) The first 3 terms of arithmetic series are $-5+1+7+\ldots$:

(i) Find the 55th term of the series. **2**

(ii) Hence, or otherwise find the sum of the first 55 terms of the series. **2**

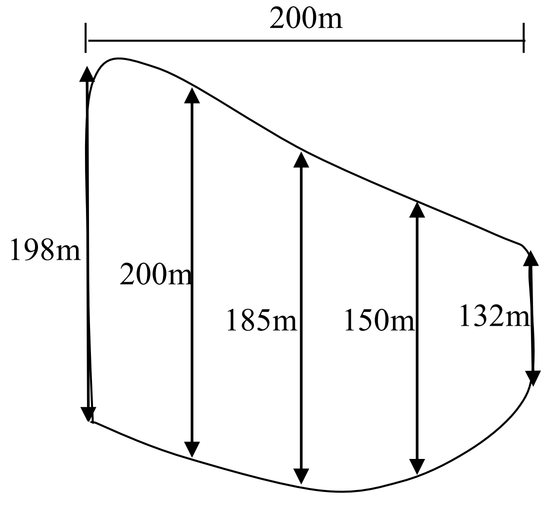
(b) Given that $\sin\theta = \frac{2}{3}$ and $90^\circ \le \theta \le 180^\circ$, find the exact value of the following expressions, making sure that you fully simplify your answers: **3**

i) $\cos\theta$.

ii) $\sin\theta - \tan\theta$.

(c) State the domain and range of the function $y = x^2 - 9$. **2**

(d) The diagram below shows an aerial view of the Duck Pond in Centennial park. The length of the Duck Pond is 200 m and the width of the lake is shown at 50 metre intervals. **3**



Use Simpson's rule with 5 function values to find an approximation for the surface area of the lake.

Question 5. (12 Marks) Use a SEPARATE Answer Sheet. **Marks**

(a) Find the values of k for which the quadratic equation $0 = x^2 - kx + k + 8$:

(i) has two equal roots. **1**

(ii) has two real roots. **2**

(b) Find all the values of θ such that $\sin 2\theta = -\frac{\sqrt{3}}{2}$ for $0 \le \theta \le 360^\circ$ **3**

(c) (i) Sketch the curve $y = x^2 - 3x + 2$, clearly indicating the x and y intercepts. **2**

(ii) On the same set of axes from part (i), sketch the line $y = 2 - x$ and shade the region represented by: **2**

$$\int_0^2 \left(2 - x - (x^2 - 3x + 2)\right) dx$$

(iii) Find the exact value of the area of the shaded region in part (ii) **2**

Question 6. (12 Marks) Use a SEPARATE Answer Sheet. **Marks**

(a) A function $f(x)$ is defined by $f(x) = x^3 - 6x^2 + 9x - 4$ for the domain $-1 \le x \le 5$.

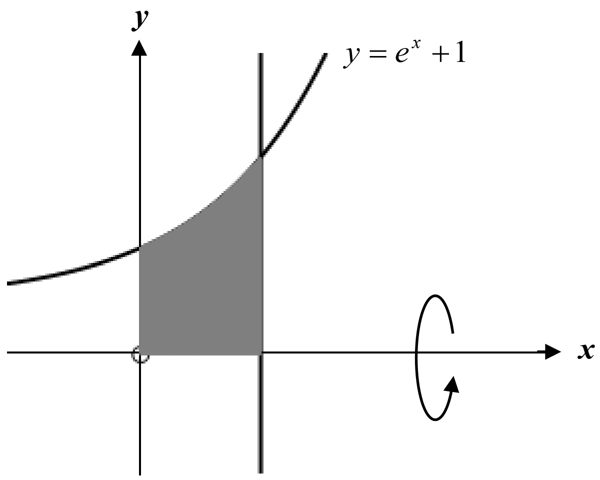
(i) Find the coordinates of the turning points of $f(x)$ and determine their nature. **3**

(ii) Find the coordinates of the point of inflexion. **2**

(iii) Hence, sketch the graph of $y = f(x)$, showing all turning points, the point of inflexion and the y-intercept. **3**

(b) Evaluate $\sum_{r=3}^{6} \frac{1}{r^2}$, to 2 decimal places. **1**

(c) **3**



In the diagram the shaded region is bounded by the curve $y = e^x + 1$, the x-axis, the y-axis and the line $x = 1$.

Find the exact volume of the solid formed when the region is rotated about the x-axis, making sure that you fully simplify your answer.

Question 7. (12 Marks) Use a SEPARATE Answer Sheet. **Marks**

(a) Let α and β be the solutions to the equation $0 = x^2 - 5x + 7$.

(i) Find $\alpha\beta$ and $\alpha + \beta$. **1**

(ii) Hence, find $\alpha^3\beta + \alpha\beta^3$. **2**

(b) A parabola is defined by the equation $-8x = y^2 - 4y - 4$.

(i) Show that the equation can be written as $(y-2)^2 = -8(x-1)$. **1**

(ii) Sketch the parabola, clearly indicating the coordinates of the vertex, the coordinates of the focus and the equation of the directrix. **4**

(c) Consider the geometric series

$$1 + (\sqrt{3} - 2) + (\sqrt{3} - 2)^2, \ldots$$

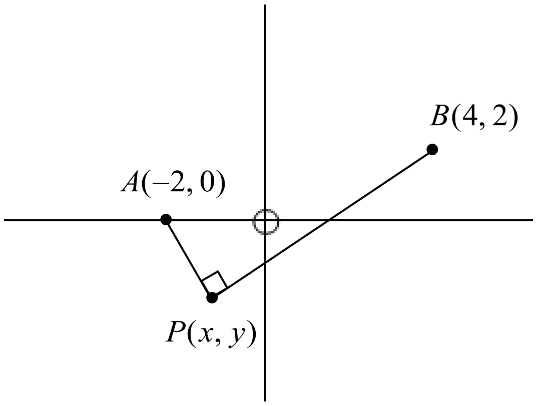
(i) Find the value of the 7^{th} term of the series to 2 significant figures. **1**

(ii) Explain why the series has a limiting sum. **1**

(iii) Find the exact value of the limiting sum of the series, making sure that you fully simplify your answer. **2**

Question 8. (12 Marks) Use a SEPARATE Answer Sheet. **Marks**

(a)



In the diagram above the points A, B, and P have coordinates (-2, 0), (4, 2) and (x, y) respectively.

(i) Given that $\angle APB = 90^\circ$, show that the locus of the point P is the curve with equation $x^2 - 2x + y^2 - 2y - 8 = 0$. **2**

(ii) Show that this is a circle and find its centre and radius. **2**

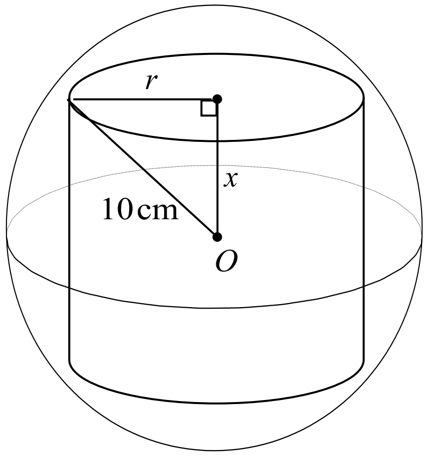
(b) Solve the equation $3^{2x} - 7(3^x) - 18 = 0$. **2**

(c) Find the coordinates of the point P on the curve $y = \dfrac{1}{(x-5)^2}$ at which the tangent to the curve is perpendicular to the line $y = 4x - 3$. **3**

(d) Differentiate $3xe^x$ and hence find $\int_0^2 e^x + xe^x \, dx$. **3**

Question 9. (12 Marks) Use a SEPARATE Answer Sheet. **Marks**

(a)



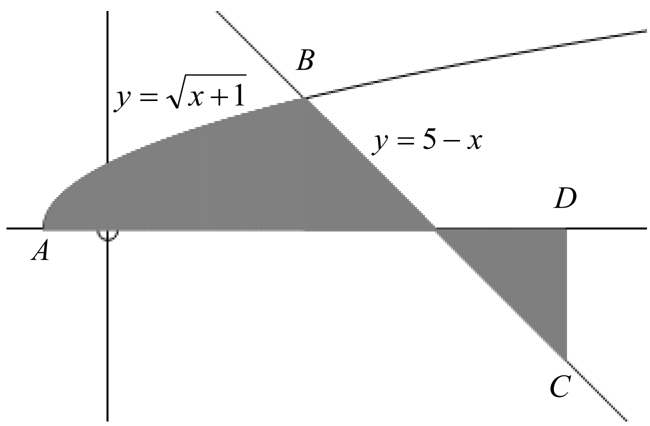
A cylinder is to be cut out of a sphere, as shown in the diagram.
The sphere has radius 10 cm and the cylinder has radius r cm and height $2x$ cm.

(i) Show that the volume, V, of the cylinder is $V = 2\pi x(100 - x^2)$ **2**

(ii) Find the value of x for which the volume of the cylinder is a maximum. You must give reasons why your value of x gives the maximum volume. **3**

(b) Find $f(x)$, if $f'(x) = e^{3x-2}$ and $f(1) = \frac{4e}{3}$. **2**

(c) The shaded region $ABCD$ is bounded by the line $x = 7$, the curve $y = \sqrt{x+1}$, the line $y = 5 - x$ and the x-axis, as in the diagram.



(i) Show that B has coordinates (3, 2). **2**

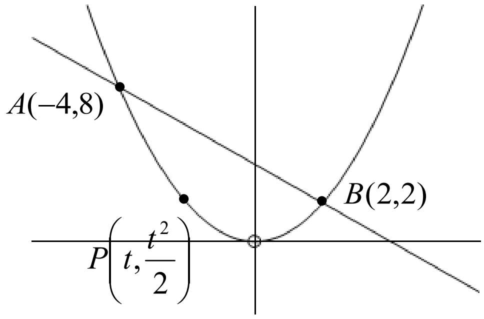
(ii) Find the shaded area. **3**

Question 10. (12 Marks) Use a SEPARATE Answer Sheet. **Marks**

(a) (i) Prove that $(1+\tan^2\theta)\sin^2\theta = \tan^2\theta$. **2**

(ii) Hence, or otherwise solve the equation **3**
$2(1+\tan^2\theta)\sin^2\theta + 5\tan\theta = 7$ for $-180^\circ \le \theta \le 180^\circ$, giving your answer to the nearest minute.

(b)



In the diagram $A(-4,8)$ and $B(2,2)$ are points of intersection of the parabola $y=\frac{x^2}{2}$ with the line $y=4-x$. The point $P\left(t,\frac{t^2}{2}\right)$ is a variable point on the parabola, below the line.

(i) Find the area between the line $y=4-x$ and the curve $y=\frac{x^2}{2}$. **3**

(ii) Show that the shortest distance between the line and the point P **1**

can be given by the expression:

$$d=\frac{4-t-\frac{t^2}{2}}{\sqrt{2}} \text{ for } -4\le t\le 2.$$

(iii) Hence, show that the maximum area of the triangle APB is $\frac{3}{4}$ of the **3**

area between the line and the curve.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

#1 a) 20.7 (to 3 sf).

b) $x-5 \le 7$ AND $x-5 \ge -7$

$x \le 12$ $\qquad$ $x \ge -2$.

$-2 \le x \le 12$.

c) $(x-5)(x+2)=0$

$x=5 \quad x=-2$.

d) $y = x^6 - 2x^{-1} + 3$

$\frac{dy}{dx} = 6x^5 + 2x^{-2}$.

e) $16 + 8\sqrt{3} + 3 = a + b\sqrt{3}$

$\therefore a = 19$

$b = 8$.

f). $6x + 4y = 28$

$-6x + 12y = +36$

$16y = 64$

$y = 4$

$3x + 8 = 14$

$3x = 6$

$x = 2$.

2a). $\frac{dy}{dx} = m(\tan) = 2x+5$

at $x = 1$ ✓

$m(\tan) = 7$ ✓

$m(\text{nomal}) = -\frac{1}{7}$ ✓ $(1, 6)$

$\therefore$ eq: $y - 6 = -\frac{1}{7}(x-1)$ 3.

$7y - 42 = -x+1$ ✓

$7y = -x + 43$. ✓

b)i) $m(AC) = \frac{2-0}{5-3} = \frac{2}{2} = 1$. ✓ 1.

ii) AC: $y - 2 = 1(x-5)$

$y - 2 = x - 5$

$y = x - 3$. ✓ 1.

$x - y - 3 = 0$.

iii). $x = 9 - 2y$ —①.

$2(9-2y) + y - 6 = 0$.

$18 - 4y + y - 6 = 0$ 2.

$-3y = -12$

$y = 4$. ✓ ②

$x = 9 - 8$

$x = 1$ ✓ $B(1, 4)$.

iv). $B(1, 4)$ $\quad \overset{a}{1}x \overset{b}{-}y \overset{c}{-3} = 0$

$\perp d = \frac{|(1)(1) + (-1)(4) + (-3)|}{\sqrt{1^2 + (-1)^2}}$ ✓ $= \left|\frac{1-4-3}{\sqrt{2}}\right|$

$= \frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{6\sqrt{2}}{2}$ ✓ 2.

$= 3\sqrt{2}$

$d(AC) = \sqrt{(5-3)^2 + (2-0)^2}$

$= \sqrt{4+4}$

$= \sqrt{8}$ ✓

3.

✓

$\therefore$ v) area of $\Delta ABC = \frac{1}{2}(\sqrt{8})(3\sqrt{2}) = 6u^2$. ✓

3a). $\int_1^2 \frac{3x^4}{x^3} - \frac{2x}{x^3}\, dx = \int_1^2 3x - 2x^{-2}$

$= \left[\frac{3x^2}{2} \quad \frac{2x^{-1}}{-1}\right]_1^2$

$= \frac{3}{2}x^2 + \frac{2}{x}\Big]_1^2$ 2

$= (6+1) - (\frac{3}{2}+2)$

$= (7) - (3\frac{1}{2})$

$= \underline{3\frac{1}{2}}$.

b). $\angle BDA = 23°$ alt $\angle =$ 2.

$\therefore \angle BCD = 70°$ coint $\angle$ suppl.

$\angle ACD = 52°$.

c) i) $y = (x^2-3)^7$ 2

$y' = 7(x^2-3)^6 \times 2x = 14x(x^2-3)^6$

ii) $y = \frac{2x}{e^x}$

$u = 2x \qquad v = e^x$ 2

$u' = 2 \qquad v' = e^x$

$y' = \frac{(e^x)(2) - (2x)(e^x)}{(e^x)^2} = \frac{2e^x(1-x)}{e^{2x}} = \frac{2(1-x)}{e^x}$

d) i) $BC^2 = 2.4^2 + 5.8^2 - 2(2.4)(5.8)\cos 30°$

$BC^2 = 15.289$ 2

$\underline{BC = 3.9}$.

ii) area $= \frac{1}{2}(2.8)(3.9)\sin 20°$

$= 1.867$ 2

$= 2\,m^2$ (to nearest m)

#4a) $-5+1+7\ldots \quad a=-5,\ d=6.$

i) $T_{55} = a+54d$

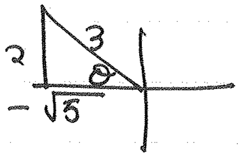
$= -5+54(6)$

$= 319$

ii) $S_{55} = \frac{55}{2}\left[2(-5)+(54)(6)\right]$

$= 8635$

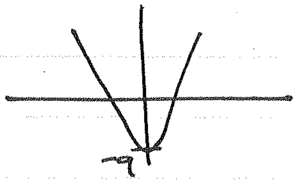
b). $\sin\theta = \frac{2}{3}\ \frac{o}{h}$



i) $\cos\theta = \frac{-\sqrt{5}}{3}$

ii) $\sin\theta - \tan\theta = \frac{2}{3} - \frac{2}{-\sqrt{5}}\frac{\sqrt{5}}{\sqrt{5}} = \frac{2}{3}+\frac{2\sqrt{5}}{5}$. ✓(simplified correctly)

c)



D: $x \in \mathbb{R}$.

R: $y \geq -9$.

d).

x	y	S	Sxy
0	198	1	198
50	200	4	800
100	185	2	370
150	150	4	600
200	132	1	132

$\sum$ 2100

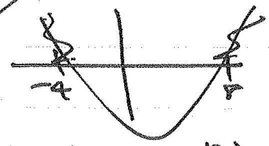
$\therefore$ Area $= \frac{50}{3} \times 2100$

$= 35000\ m^2$

#5 a). for equal roots $\Delta = 0$

$$(-k)^2 - 4(1)(k+8) = 0$$
$$k^2 - 4k - 32 = 0$$
$$(k-8)(k+4) = 0$$
$$k = 8 \quad k = -4.$$

for real roots $\Delta \geq 0$



$k \leq -4 \quad k \geq 8.$

(forget equals sign then 1 mark only)

b) $\sin 2\theta = -\frac{\sqrt{3}}{2}.$

rel $\angle = 60^\circ$

sin is $-$ in

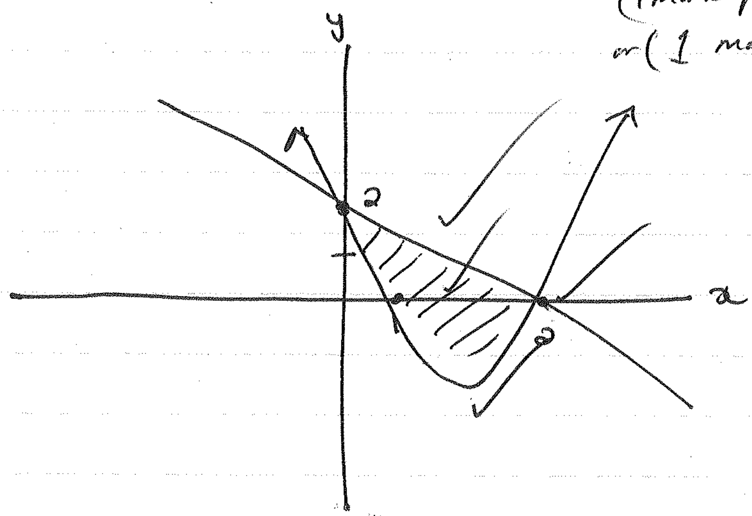
3rd	4th.
$2\theta = 240^\circ, 600$	$2\theta = 300^\circ, 660$
$\theta = 120, 300$	$\theta = 150^\circ, 330^\circ$

(correct domain)

(1 mark for partial soln, or (1 mark for rel. $\angle$)

c) $y = x^2 - 3x + 2.$

$y = (x-2)(x-1)$



$$A = \int_0^2 (2 - x - x^2 + 3x - 2)\,dx$$
$$= \int_0^2 2x - x^2 \,dx$$
$$= \left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2$$
$$= \left(4 - \frac{8}{3}\right) - (0)$$
$$= \frac{4}{3}u^2.$$

#6. $y = x^3 - 6x^2 + 9x - 4 \qquad -1 \le x \le 5$

S.P. $y' = 0$: $3x^2 - 12x + 9 = 0$. 6. 3.2.

$(x-1)(x-3) = 0$

$x = 1$	$x = 3$
$y = 0$	$y = -4$.

nature: $y'' = 6x - 12$

at $x = 1$ at $x = 3$

$y'' = -6$ $y'' = 6$

max.   min

$(1, 0)$ max $(3, -4)$ min

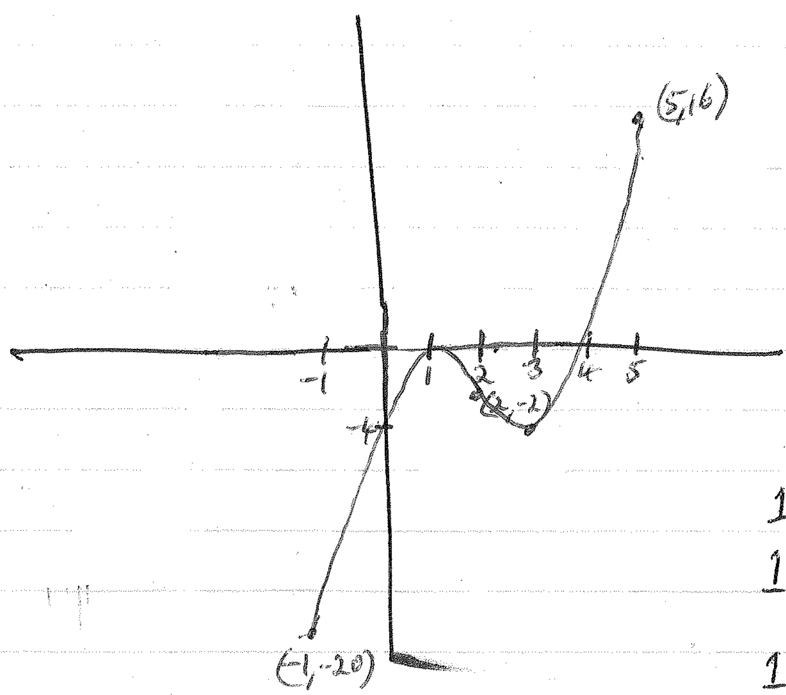
P.O.I. $y'' = 0$

$x = 2$ check (or similar)

$y = -2$ change in concavity

x	0	2	3
y''	-12	0	6

$x = -1$	$x = 5$
$y = -20$	$y = 16$



1 - end points

1 - max, min & poI

1 - shape + direct of curve.

#6b) $\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36} \approx 0.24$ (to 2 dp).

c)

$$V = \pi\int_0^1 (e^x+1)^2\,dx$$

$$= \pi\int_0^1 e^{2x}+2e^x+1\,dx$$

$$= \pi\left[\frac{e^{2x}}{2}+2e^x+x\right]_0^1$$

$$= \pi\left[\left(\frac{e^2}{2}+2e+1\right)-\left(\frac{1}{2}+2\right)\right]$$

$$= \pi\left(\frac{e^2}{2}+2e+1-2\tfrac{1}{2}\right)$$

$$= \pi\left(\frac{e^2}{2}+2e-1\tfrac{1}{2}\right)\ u^3.$$

#7a). $x^2-5x+7=0$ 1.

$\alpha+\beta=5$ $\alpha\beta=7$

(i) $\alpha^3\beta+\alpha\beta^3$

$= \alpha\beta(\alpha^2+\beta^2)$

$= \alpha\beta\left((\alpha+\beta)^2-2\alpha\beta\right)$ 2.

$= 7(5^2-2\times7)$

$= 7(25-14)$

$= 77$.

7b i). $-8x+4 = y^2-4y$

$-8x+4+(2)^2 = y^2-4y+(2)^2$

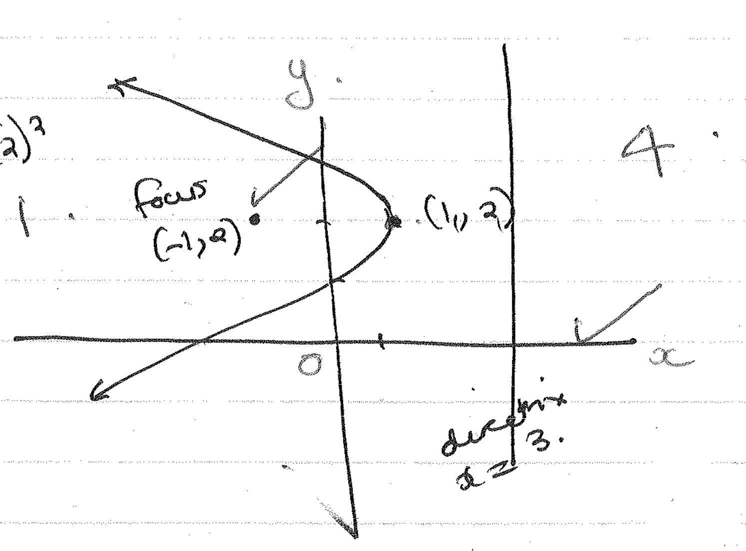
$(y-2)^2 = -8x+8$ 1.

$(y-2)^2 = -8(x-1)$

$(y-2)^2 = -4(2)(x-1)$

$\therefore$ vertex $(1,2)$

Focal length $=2$.



4

7 c) i) $1 + (\sqrt{3}-2) + (\sqrt{3}-2)^2 \ldots$

$T_7 = ar^6$

$= 1(\sqrt{3}-2)^6$

$= 0.00657$

1.

ii) $r = \sqrt{3}-2 \approx -0.267$

$-1 < r < 1$

1.

iii) $S_\infty = \frac{a}{1-r}$

$= \frac{1}{1-(\sqrt{3}-2)} = \frac{1}{1-\sqrt{3}+2} = \frac{1}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$

$= \frac{3+\sqrt{3}}{9-3}$

2.

$= \frac{3+\sqrt{3}}{6}$.

#8 $m(AP) = \frac{y-0}{x+2}$ $m(BP) = \frac{y-2}{x-4}$

$\therefore AP \perp BP$: $\frac{y}{x+2} = \frac{-(x-4)}{y-2}$

$y(y-2) = -(x-4)(x+2)$

$y^2 - 2y = -x^2 + 2x + 8$

$x^2 - 2x + y^2 - 2y - 8 = 0$.

ii) $x^2 - 2x + (1)^2 + y^2 - 2y + (1)^2 = 8 + (1)^2 + (1)^2$

$(x-1)^2 + (y-1)^2 = 10$.

$\therefore$ circle

with centre $(1, 1)$

radius $\sqrt{10}$.

b) $3^{2x} - 7.3^x - 18 = 0$

let $3^x = k$

$k^2 - 7k - 18 = 0$

$(k-9)(k+2) = 0$

$k = 9$	$k = -2$
$3^x = 3^2$	$3^x = -2$
$x = 2$	no soln

8c) $y = (x-5)^{-2}$ $\perp$ $y = 4x-3$

$m = 4.$

$y' = -2(x-5)^{-3} \cdot 1$

$= \dfrac{-2}{(x-5)^3}$

$\dfrac{-2}{(x-5)^3} = -\dfrac{1}{4}$

$8 = (x-5)^3$

$\sqrt[3]{8} = x-5$

$x-5 = 2$

$x = 7$

$y = 2^{-2} = \frac{1}{4}.$ $\therefore$ $P(7, \frac{1}{4})$

d). let $y = 3xe^x$

$u = 3x$ $v = e^x$

$u' = 3$ $v' = e^x$

$\dfrac{dy}{dx} = 3e^x + 3xe^x$

$\int \dfrac{dy}{dx} = \int 3(e^x + xe^x)\,dx$

$\int_0^2 e^x + xe^x\,dx = \frac{1}{3}\left[3xe^x\right]_0^2$

$= \boxed{\left[xe^x\right]_0^2}$

$= 2e^2 - 0$

$= 2e^2.$

#9a) $V_{\text{of cylinder}} = \pi r^2 \times h.$

$= \pi r^2 \times 2x$

by pythag: $r^2 + x^2 = 100.$

$r^2 = 100 - x^2.$

$\therefore V = 2\pi x(100 - x^2).$

To max vol: $V' = 0.$

$V = 200\pi x - 2\pi x^3$

$V' = 200\pi - 6\pi x^2 = 0.$

$200\pi = 6\pi x^2$

$\frac{200\cancel{\pi}}{6\cancel{\pi}} = x^2$

$x = \sqrt{\frac{100}{3}}$

check max: $V'' = -12\pi x < 0.$

max.

b). $f'(x) = e^{3x-2}$

$f(x) = \frac{e^{3x-2}}{3} + c$

$f(1) = \frac{e^{3-2}}{3} + c = \frac{4e}{3}$

$c = \frac{4e}{3} - \frac{1e}{3} = e.$

$\therefore f(x) = \frac{e^{3x-2}}{3} + e$

#9c). $\sqrt{x+1} = 5-x$

$x+1 = 25 - 10x + x^2$

$0 = x^2 - 11x + 24$

$0 = (x-3)(x-8)$

$x = 3$ $\quad \cancel{x=8}$.

$y = 2$. $\quad B(3,2)$

or any alternative method.

$$\text{Shaded area} = \int_{-1}^{3}(x+1)^{\frac{1}{2}} + \int_{3}^{5}(5-x)\,dx + \left|\int_{5}^{7}(5-x)\,dx\right|$$

$$= \frac{2}{3}(x+1)^{\frac{3}{2}}\Big]_{-1}^{3} + 5x - \frac{x^2}{2}\Big]_{3}^{5} + \left|5x - \frac{x^2}{2}\Big]_{5}^{7}\right|$$

$$= \frac{2}{3}\sqrt{(x+1)}^{\,3}\Big]_{-1}^{3} + \left(25 - 12\tfrac{1}{2}\right) - \left(15 - \frac{9}{2}\right) + \left(35 - \frac{49}{2}\right) - \left(25 - \frac{25}{2}\right)$$

$$= \frac{2}{3}\left[8 - 0\right] + \left(12\tfrac{1}{2}\right) - \left(\frac{21}{2}\right) + \left|\frac{21}{2} - \frac{25}{2}\right|$$

$$= \frac{16}{3} + 2 + |-2|$$

$$= \frac{16}{3} + 4$$

$$= 9.\dot{3}\ u^2$$

#10. LHS:

$(\sec^2\theta)\sin^2\theta$

$= \frac{1}{\cos^2\theta}\cdot\sin^2\theta$

$= \tan^2\theta$

$=$ RHS.

$2\tan^2\theta + 5\tan\theta - 7 = 0$ $\quad$ 14. 7.2.

$(2\tan\theta + 7)(\tan\theta - 1) = 0$.

$\tan\theta = -\frac{7}{2}$ $\quad$ rel $\angle = 74°3'$ $\quad\quad$ $\tan\theta = 1$ $\quad$ rel $\angle = 45°$

2nd	4th	1st	3rd
$\theta = 105°57'$	$\theta = -74°3'$	$\theta = 45°$;	$\theta = 225°$ / $-135°$

10b). $A = \int_{-4}^{2} (4-x) - \left(\frac{x^2}{2}\right) dx$

$= \left[4x - \frac{x^2}{2} - \frac{x^3}{6}\right]_{-4}^{2}$

$= \left(8 - 2 - \frac{8}{6}\right) - \left(-16 - 8 + \frac{64}{6}\right)$

$\frac{14}{3} - \frac{-40}{3}$

$= 18$

ii) $P\left(t, \frac{t^2}{2}\right)$ $\quad 1x + y - 4 = 0$.

$\perp d = \frac{|(1)(t) + (1)\left(\frac{t^2}{2}\right) + (-4)|}{\sqrt{1^2+1^2}}$

$\perp d = \frac{|t + \frac{t^2}{2} - 4|}{\sqrt{2}}$

since $-4 \le t \le 2$. and d must be positive (or similar)

$\perp d = \frac{-t - \frac{t^2}{2} + 4}{\sqrt{2}}$

iii) $A_\Delta = \frac{1}{2}(AB) \perp d$.

$d(AB) = \sqrt{(-4-2)^2 + (8-2)^2}$

$= \sqrt{36 + 36}$

$= \sqrt{72}$

$= 6\sqrt{2}$

$= \frac{1}{2} 6\cancel{\sqrt{2}} \times \frac{\left(4 - t - \frac{t^2}{2}\right)}{\cancel{\sqrt{2}}}$

$= 12 - 3t - \frac{3t^2}{2}$.

$\therefore$ max area $A' = 0$: $-3 - 3t = 0$.

$-3 = 3t$

$-1 = t$

$\therefore A_\Delta = 12 - 3(-1) - \frac{3(-1)^2}{2}$

$= 12 + 3 - 1\frac{1}{2}$

$= 13.5$

and 13.5 is $\frac{3}{4}$ of 18.